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6. AUTHOR(S)

Harry F. Tiersten

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Rensselaer Polytechnic Institute
Dept. of Mechanical Engineering
Troy, NY 12180

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An analysis of the in-plane acceleration sensitivity of ST-cut quartz surface wave resonators with the substrate extending beyond interior rectangular supports was performed.

An analysis of the normal acceleration sensitivity of contoured AT- and SC-cut quartz crystal resonators supported along rectangular edges was performed.

In subsequent work it was shown analytically that the normal acceleration sensitivity of contoured quartz resonators with rectangular supports vanishes when the centers of the mode shape and support rectangle coincide.

It has further been shown that the resultant in-plane acceleration sensitivity of contoured quartz resonators with rectangular supports vanishes when the centers of the support rectangle and the mode shape coincide and the active resonator region is supported symmetrically on both sides.

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ABSTRACT CONTINUED

Since the resultant in-plane acceleration sensitivity vanishes when the centers of the support rectangle and the mode shape coincide, an analysis of the influence of a relative displacement of the center of the mode shape with respect to rectangular supports was performed.

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1. "On the Normal Acceleration Sensitivity of ST-Cut Quartz Surface Wave Resonators Supported Along Rectangular Edges," H. F. Tiersten and D. V. Shick, Journal of Applied Physics, 64, 4334-4341 (1988).
2. "An Analysis of the In-Plane Acceleration Sensitivity of ST-Cut Quartz Surface Wave Resonators with the Substrate Extending Beyond the Supports," D. V. Shick and H. F. Tiersten, Proceedings of the 42nd Annual Symposium on Frequency Control, U.S. Army Electronics Technology and Devices Laboratory, Fort Monmouth, New Jersey and Institute of Electrical and Electronics Engineers, New York, IEEE Catalog Number 88CH2588-2, 230-238 (1988).
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4. "An Analysis of the Normal Acceleration Sensitivity of Contoured Quartz Resonators Rigidly Supported Along the Edges," H. F. Tiersten and D. V. Shick, 1988 Ultrasonics Symposium Proceedings, IEEE Catalog Number 88CH2578-3, Institute of Electrical and Electronics Engineers, New York, 357-363 (1988).
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9. "An Analysis of the Normal Acceleration Sensitivity of Contoured Quartz Resonators with Simple Rectangular Supports," Y. S. Zhou, D. V. Shick and H. F. Tiersten, 1989 Ultrasonics Symposium Proceedings, IEEE Catalog Number 89CH2791-2, Institute of Electrical and Electronics Engineers, New York, 393-400 (1989).
10. "On the Influence of a Fabrication Imperfection on the Normal Acceleration Sensitivity of Contoured Quartz Resonators with Rectangular Supports," Y. S. Zhou and H. F. Tiersten, Proceedings of the 44th Annual Symposium on Frequency Control, U.S. Army Electronics Technology and Devices Laboratory, Fort Monmouth, New Jersey and Institute of Electrical and Electronics Engineers, New York, IEEE Catalog Number 90CH2818-3, 452-460 (1990).
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14. "On the In-Plane Acceleration Sensitivity of Contoured Quartz Resonators Supported Along Rectangular Edges," H. F. Tiersten and Y. S. Zhou, to be published in the Journal of Applied Physics (1991). Page proofs have been read.
15. "The Increase in the In-Plane Acceleration Sensitivity of the Plano- Convex Resonator Due to its Thickness Asymmetry," H. F. Tiersten and Y. S. Zhou, to be published in the Proceedings of the 45th annual Symposium on Frequency Control, U.S. Army Electronics Technology and Devices Laboratory, Fort Monmouth, New Jersey and Institute of Electrical and Electronics Engineers, New York (1991).
16. "An Analysis of the Normal Acceleration Sensitivity of Contoured Quartz Resonators Stiffened by Identical Top and Bottom Quartz Cover Plates Supported by Clips," Y. S. Zhou and H. F. Tiersten, to be published in the Proceedings of the 45th Annual Symposium on Frequency Control, U.S. Army Electronics Technology and Devices Laboratory, Fort Monmouth, New Jersey and Institute of Electrical and Electronics Engineers, New York (1991).

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT:

Harry F. Tiersten
Douglas V. Shick - Research Associate
Yan Sheng Zhou - Research Associate

BRIEF OUTLINE OF RESEARCH FINDINGS

An analysis of the in-plane acceleration sensitivity of ST-cut quartz surface wave resonators with the substrate extending beyond interior rectangular supports was performed[5,8]. This structure constitutes a revised version of the frit support configuration used by Raytheon and is intended to reduce the in-plane acceleration sensitivity. In the treatment the variational principle with all natural conditions was extended in such a way as to permit interior surfaces of discontinuity with spring supports at which the approximating solution functions need not satisfy the constraint-type conditions. This extension is required for the approximation procedure used in the calculation of the biasing state. The calculated in-plane acceleration sensitivity indicates how to select the distances between the supports so as to minimize the in-plane acceleration sensitivity for a given surface wave resonator.

An analysis of the normal acceleration sensitivity of contoured AT- and SC-cut quartz crystal resonators supported along rectangular edges was performed[7,9]. The variational principle with all natural conditions for anisotropic static flexure obtained in earlier work was used in the calculation of the flexural biasing state. However, in this work[7,9] the biasing shearing stresses with quadratic variation across the thickness of the plate were determined _a posteriori_ in the conventional manner. The accompanying quadratically varying biasing shearing strains, which are very important in this work, were obtained from the constitutive equations along with all other quadratically varying strains resulting from the anisotropy. The calculated biasing deformation fields were employed in an existing perturbation equation along with the equivalent trapped energy mode shapes of the contoured resonator to calculate the normal acceleration sensitivity. It was shown[7,9] that the normal acceleration sensitivity vanishes for many cases and is less than a few parts in 10^{13} per g for all cases considered and is usually less than a few parts in 10^{15} per g, all of which are negligible. Furthermore, it was noted[7,9] that the calculated results were always very close to the accuracy of the calculation, which indicates that the results are not reliable.

In subsequent work[10,12] it was shown analytically that the normal acceleration sensitivity of contoured quartz resonators with rectangular supports vanishes when the centers of the mode shape and support rectangle coincide. This result is a consequence of symmetry and applies to any shape possessing the required symmetry. This analytical result indicates that the results presented in Refs. 7 and 9 are simply plots in the round off error and nothing more. Since it is essentially impossible to realize perfect symmetry in practice, an analysis of the influence of a mispositioning of the center of the support rectangle with respect to the center of the mode shape on the normal acceleration sensitivity was performed. It was shown[10,12] that for a small mispositioning of the centers the normal acceleration sensitivity increases linearly with the mispositioning dimension. This linear increase with mispositioning

dimension underscores the importance of care in fabrication to achieve low acceleration sensitivities. An orientation of the support configuration for a particular aspect ratio of an SC cut, for which the linear increase in the normal acceleration sensitivity with the mispositioning dimension exhibits a strong minimum, was found.

It has further been shown[11,14] that the resultant in-plane acceleration sensitivity of contoured quartz resonators with rectangular supports vanishes when the centers of the support rectangle and the mode shape coincide and the active resonator region is supported symmetrically on both sides. This was shown analytically and is a consequence of the symmetry of the biasing state and the mode shape. The analytical demonstration holds for any support configuration which exhibits the required symmetry with respect to the center of the mode shape. It was also shown[11,14] that when the active region is supported on one side only, in-plane flexure is induced in the plate and this results in an in-plane acceleration sensitivity of a few parts in 10^{10} per g.

Since the resultant in-plane acceleration sensitivity vanishes when the centers of the support rectangle and the mode shape coincide, an analysis of the influence of a relative displacement of the center of the mode shape with respect to rectangular supports was performed[13]. The calculated influence of the mispositioning of the centers in the in-plane case[13] turned out to be an order of magnitude smaller than in the normal case[10,12]. Consequently, the aforementioned orientation of an SC cut with the strong minimum in the normal case[10,12] is indeed practical. The fact that the resultant in-plane acceleration sensitivity vanishes for a perfectly symmetric combined resonator plus support system[11,14] indicates that a biconvex resonator will have lower resultant in-plane acceleration sensitivity than a plano-convex one because of its inherent additional symmetry. Since a plano-convex resonator is easier to fabricate, an analysis of the degradation in the in-plane acceleration sensitivity that arises from the contour being on one side only was performed[15]. The calculated results show[15] that the in-plane acceleration sensitivity increases with decreasing radius of curvature and lies in the range of a few parts in 10^{12} per g, which is quite small.

Since it is essentially impossible to construct a perfectly symmetric resonator plus support configuration in practice, a stiffened structure was considered[16], which reduces the biasing deformation in the active region and provides isolation from the unavoidable variations in the details of the mounting devices. The structure consists of the active biconvex quartz resonator attached to identical top and bottom quartz cover plates by means of small sidewalls around the periphery. The mounting clips are attached to the top and bottom cover plates without touching the active plate. An analysis of the normal acceleration sensitivity of this configuration was

performed[16]. The variational principle for anisotropic static flexure in which all conditions arise as natural conditions was extended to include the influence of the cover plates, the active region and the clips. The calculations reveal that increasing the thickness of the cover plates relative to the active plate reduces the normal acceleration sensitivity arising from either a misplacement of a clip or a mispositioning of the center of the mode shape. The calculations also showed that the orientation of the support configuration for a particular aspect ratio of an SC cut which yielded a strong minimum in the mispositioning coefficients for normal acceleration sensitivity without cover plates also yields a strong minimum when cover plates are present.